Three-dimensional frequency-domain full waveform inversion with phase encoding
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SUMMARY
Three-dimensional quantitative seismic imaging methods such as full waveform inversion, still suffer from their prohibitive computational cost. The computational burden mainly results from the computational cost of the forward problem, namely, the seismic full wavefield modeling, that must be performed for a huge number of sources at each iteration of the inversion. Building super-shots by source assembling allows to mitigate the number of simulations per iteration but adds artifacts in the imaging because of interferences between individual shots of the super-shot. These artifacts can be mitigated by using phase encoding technique which consists of applying a deterministic or random phase shift to each individual source at each iteration of the inversion. We have implemented source assembling with phase encoding in acoustic frequency-domain fullwaveform inversion and have assessed the performances of the method with a 2D and 3D synthetic case study corresponding to the SEG/EAGE overthrust model.

INTRODUCTION
Because 3D prestack depth imaging methods such as prestack depth migration and full waveform inversion (FWI) are computationally expensive, the simultaneous-shot technique (Capdeville et al., 2005) can provide an interesting compromise between computational efficiency and imaging accuracy. Taking advantage of the linear relationship between the seismic wavefield and the source, the simultaneous-shot technique consists of assembling individual sources to mitigate the number of seismic wavefield modeling performed during the imaging. The computation cost of one migration or FWI iteration is reduced proportionally to the number of shots stacked in each shot assemble super-shot. However, the imaging is altered by artifacts associated with the interference between the individual shots of a given super-shot, that may require additional iterations of the inversion before convergence towards an acceptable model. These artifacts can be reduced by encoding specific phase for each shot in the super-shot. This technique, called phase encoding (PE) (Morton and Ober, 1998; Jing et al., 2000; Romero et al., 2000), was originally proposed for prestack migration. Since the imaging kernels of prestack migration and frequency-domain FWI are basically the same, the technique can be used without any particular modification for FWI. We have implemented the simultaneous-shot technique with phase encoding in frequency-domain FWI. In a previous study, we assessed the performances of the method when only a limited number of shots are assembled with random PE within several super-shots (Ben-Hadj-Ali et al., 2009). In this study, we tested the method when all the sources are assembled to form one single super-shot. We applied the method to a realistic synthetic case study using the SEG/EAGE overthrust model and assessed the number of iterations required to obtain an acceptable velocity model.

FREQUENCY-DOMAIN FULL-WAVEFORM INVERSION WITH PHASE ENCODING
FWI is generally recast as an iterative local optimization based on the minimization of the least-squares objective function (Pratt et al., 1998) given by,

$$ C^k(m) = \left( d_o - d^k_c(m) \right) \left( d_o - d^k_c(m) \right)^* $$

(1)

where $m$ is the model, $d_o$ the recorded data and $d^k_c(m)$ the modeled (computed) data at iteration $k$. The model perturbation, based upon the Born approximation and the steepest descent method, is given by

$$ \delta m^k = -\alpha^k \left( diag H^k_0 + \gamma^k Id \right)^{-1} C^k $$

(2)

where $diag H^k_0$ is the diagonal of the approximate or pseudo Hessian (Shin et al., 2001), $G^k$ is the gradient, $Id$ is the identity matrix, $\alpha^k$ is the step length and $\gamma^k$ is a prewhitening factor that prevents numerical instabilities. The gradient can be efficiently computed by the adjoint-state method (Plessix, 2006). In the frequency domain, the contribution of one shot to the gradient is given by the product of the incident wavefield emitted by the shot with the backpropagated (i.e., conjugate) residual wavefields and with a sparse radiation-pattern matrix $W$, the coefficients of which depend on the parameter class ($P$-wave velocity, density, impedance, ...). The gradient is simply formed by stacking the contribution of all the shots, that gives,

$$ G^k = - \sum_{\text{shots}} P^\dagger W A^{-1} \left( d_o - d^k_c \right)^* $$

(3)

where $P$ is the incident wavefield and $A$ is the impedance matrix, the forward problem operator (Pratt et al., 1998). $^\dagger$ denotes transpose and $^*$ conjugate.

Simultaneous-shot technique
The computational burden in FWI resulting from multi-source simulations can be mitigated by assembling sources in a super-shot. The computational saving during one FWI iteration is, ideally, proportional to the number of shots assembled into the super-shot. However, this shot assemblage introduces artifacts in the gradient and Hessian estimations, that result from cross-talk effects between different shots of a super-shot. We illustrate our purpose with the gradient $G$ of a super-shot $S$ composed of 2 shots $S_1$ and $S_2$. $R_1$ and $R_2$ are the backpropagated residual wavefields associated with shots $S_1$ and $S_2$, respectively. Note that the same reasoning could apply for the Hessian. The gradient can be schematically written as
\[ G = \text{SWR}^* \]  
\[ = [S_1 \text{WR}^*_1 + S_2 \text{WR}^*_2] + [S_1 \text{WR}^*_1 + S_2 \text{WR}^*_1] \]

where \((S=S_1+S_2; R=R_1+R_2)\) in virtue of the superposition principle.

The first bracketed term corresponds to the standard gradient formed by stacking the contribution of each individual shot while the second term corresponds to cross-talk noise between shots \(S_1\) and \(S_2\). This extra term alters the imaging result. Minimization of related artifacts is achieved by the so-called PE technique.

**Phase encoding technique**

Encoding each shot of a super-shot with an arbitrary weight should reduce interference artifacts. Encoding weights \(a_1\) and \(a_2\) can be expressed with a phase term such that \(3|a_i| = |\exp(i\phi_i)| = 1, i = 1, 2\), and \((S=a_1S_1+a_2S_2; R=a_1R_1+a_2R_2)\). Equation (4) becomes

\[ G = [S_1 \text{WR}^*_1 + S_2 \text{WR}^*_2] + [a_1a_2S_1 \text{WR}^*_2 + a_1^2a_2S_2 \text{WR}^*_2]. \]  

Of note, the phase terms impact only the cross-talk terms and the PE strategy should minimize the second bracketed term in equation (5) by mean of the judicious choice of the phases \(\phi_i\). Several phase encodings have been proposed such as deterministic PE (Jing et al., 2000) and random PE (Morton and Ober, 1998; Romero et al., 2000). Random PE (RPE) generates random phases within the interval \([0, 2\pi]\). During the summation of the cross-talk terms in equation 5, the cross-talk noise tends to stack incoherently, and, hence is mitigated. Below, we will focus on the RPE technique, which has shown to be the most effective in frequency-domain FWI (Ben-Hadj-Ali et al., 2009).

**APPLICATIONS TO THE SEG/EAGE OVERTHrust VELOCITY MODEL**

**SEG/EAGE overthrust velocity model: 2D application**

We first designed a two-dimensional experiment using a 2.5-D velocity model (laterally invariant in the y-direction) and an infinite line source in the y direction to mitigate the FWI computational burden (Ben-Hadj-Ali et al., 2008). We applied 3D FWI to a dip section extracted from the SEG/EAGE overthrust velocity model (Figure 1(a)), discretized on a 801 \(\times\) 187 grid with a grid spacing \(h = 25\) m. For the 3D application, the dip section of the overthrust model was duplicated 3 times in the y direction leading to a 3D 801 \(\times\) 3 \(\times\) 187 finite-difference grid. PML absorbing boundary conditions were set on the 4 edges of the 2D model while mirroring conditions were implemented in the y direction to mimic an infinite medium. The starting model for inversion was obtained by smoothing the true velocity model with a Gaussian function of horizontal and vertical correlation lengths of 500 meters (Figure 1(b)). For a stable inversion, the true velocity structure was set in the first 100 meters of the starting model. We inverted sequentially seven frequencies ranging from 3.5 to 20 Hz. For each frequency, we computed fifteen iterations. The 2D acquisition geometry consists of a line of 200 shots and receivers equally-spaced on the surface. The final FWI model obtained without shot assembling is shown in Figure 1(c), and will be used as a reference result.

Next, all 200 shots are gathered in 1 super-shot. The final FWI model obtained without PE is shown in Figure 2. The number of iterations per frequency inversion was increased to one hundred. The inversion did not converge towards an acceptable velocity model. This failure may be explained by two reasons: first, a super-shot composed with closely-spaced sources is equivalent to an horizontal plane-wave source in virtue of the Huygen’s principle, that prevents a sufficiently-broad aperture illumination required to obtain well-resolved image of complex structures. Second, the cross-talk terms are not efficiently mitigated if no PE is used.

The FWI model obtained for one super-shot and RPE is shown in Figure 3(b). Two hundreds iterations were performed per frequency inversion. Random phase encoding is regenerating at each inversion iteration. The RPE significantly improved the final FWI model, although some artifacts remain in the low velocity layer at 1 km depth in the right hand side of the model. Figure 3(a) shows the FWI model after 50 iterations of the 7\(^{th}\) (final) frequency. Comparison between the FWI models after 50 and 200 iterations of the 7\(^{th}\) frequency shows how the FWI iterations, each of them is performed with a new random phase encoding, help to mitigate the cross-talk terms, in addition to the stack of the encoded cross-talk terms performed during each inversion iteration.

We then applied FWI to groups of frequencies, where a group of frequencies defines a set of frequencies that are simultaneously inverted. The seven frequencies ranging between 3.5 Hz and 20 Hz were involved in the inversion. We considered 4 groups of frequencies, where each group gathers two frequencies except the last one, and there is no overlap between next groups. One super-shot with RPE was inverted. The random phase encoding was regenerated for each frequency of one group and at each iteration of one group inversion. The final FWI model, obtained after 200 iterations, is similar to the FWI model obtained without source assembling (Figure 4(b)) and closely matches the true model (Figure 1(b)). This is further confirmed by the comparison between velocity logs extracted from the true model and the final FWI model (Figure 5). The FWI model after 50 iterations of the 4\(^{th}\) frequency group is also shown in Figure 4(a) for comparison. Comparison between the FWI models of Figures 3(b) and 4(b) highlights the significant effect of the simultaneous inversion of two frequencies.

We obtained a speedup of 7, where the speedup is the ratio between the elapsed times required to perform FWI without and with source assembling. Ideally, we should have reached a speed-up of 15 according to the number of wavefield solutions to be computed (15 iterations and 200 shots \textit{versus} 200 iterations and 1 shot). However, a significant amount of computations in our frequency-domain modeling method based on a hybrid direct-iterative solver (Sourbier et al., 2008) are independent of the number of shots. A better speedup should be obtained for modeling methods, the computational cost of which linearly increases with the number of shots such as time-domain method or frequency-domain method based on iterative solvers.
Figure 1: a) True velocity model. b) Starting velocity model. c) FWI velocity model without source assembling.

Figure 2: FWI velocity model with source assembling but without RPE.

Figure 3: FWI velocity model with source assembling and RPE for the last-frequency inversion. a) after 50 iterations. b) after 200 iterations. The seven frequencies were inverted successively.

Figure 4: FWI velocity model with source assembling and RPE for the last-frequency-group inversion. a) after 50 iterations. b) after 200 iterations. Four frequency groups were inverted.

Figure 5: Velocity logs extracted from the true model (blue) and from the FWI models of Figures 1(c) (black), 4(b) (green) at a) 6 kms. b) 16 kms.

SEG/EAGE overthrust velocity model: 3D application

We performed an application of 3D FWI with source assembling and RPE to the SEG/EAGE overthrust velocity model. We consider a coarse wide-aperture/wide-azimuth acquisition geometry composed of 64x64=4096 sources and receivers. Source and receiver spacings are 300 m. The true and starting FWI velocity models are shown in Figures 6(a) and 6(b) respectively. We inverted only one frequency of 3.5 Hz. The finite-difference (FD) grid was 31x131x131. The FWI models, using source assembling and RPE, are shown in Figures 7(b) and 7(c) after 100 and 300 iterations, respectively. The FWI model obtained without source assembling is shown in Figure 7(a) after 10 iterations. As in the 2D case, RPE reduces very well artifacts over iterations because random phase encoding is regenerated at each iteration. The final FWI model of 7(c) is even slightly better than the FWI model obtained without source assembling, probably, because not enough iterations were performed to obtain the FWI model without source assembling. Apart from computational efficiency, another advantage of the simultaneous-shot technique is the interactivity. Indeed, one FWI iteration lasts few minutes with the simultaneous-shot technique whereas it lasts few hours when dealing with all shots (speedup of 450). This makes the quality control of the FWI easier and flexible.

CONCLUSION

Full-shot assembling coupled with random phase encoding reveals promising to design 3D efficient frequency-domain full-waveform inversion. The efficiency of the method was proven either on 2D and 3D applications. However, the method should still tested in more realistic configurations including free-surface effects and noise issues.
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Figure 6: (a) True velocity model. (b) Starting velocity model.

Figure 7: (a) FWI velocity model without source assembling. (b) FWI velocity model with source assembling and RPE after 100 iterations. (c) FWI velocity model with source assembling and RPE after 300 iterations.
EDITED REFERENCES
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